An integrated approach for robust inventory routing problem in a three-echelon distribution system

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Abstract: This paper introduces the robust inventory routing problem with finite time horizon in three-level distribution systems considering uncertain demand and transportation cost. The vendor is responsible for replenishing distribution centres and distribution centres replenish geographically scattered customers. The products are distributed by capacitated vehicles and, depending on the decision variables, multiple vehicles are assigned to each distribution centre. The inventories are kept both in distribution centres and customer sites. The objective is was to find a combined transportation and inventory strategy and minimise system cost while meeting the demand of each customer without shortage and ensuring feasibility regardless of the realised demands and transportation cost. The proposed system is integrated by a mixed integer linear programming (MILP) formulation for deterministic case of the problem. Moreover, the corresponding robust counterpart is formulated with regard to three different techniques of box, polyhedral and interval-polyhedral and analysed using adjustable uncertainty parameters on a test bed. Finally, to cope with intractability of large size problems, an imperialist competitive algorithm is developed by genetic algorithm operators.

Keywords: inventory routing problem; IRP; distribution; logistics; robust optimisation; imperialist competitive algorithm; ICA.

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1 Introduction

Coordination plays an important role in successful performance of all parts of supply chain. Since all parts of a supply chain are tightly connected, their coordination results in minimising system cost and also better response performance for customers' requirements. Specifically, in distribution logistics, by considering integrity, producers and purchasers are not allowed to independently make decisions. Distribution problem as transportation problem and inventory management have been investigated by many researchers as inventory routing problem (IRP). The main body of IRP can be defined as solving three questions of how much to deliver, when to deliver and in what order to deliver. Features of IRPs can be categorised in terms of time horizon, topology, demand, routing, inventory policy, inventory decision, fleet composition and fleet size (Coelho et al., 2012) In real world situations, IRPs arise in various industries such as maritime logistics (Song and Furman, 2010), blood distribution (Hemmelmayr et al., 2009), gas companies (Uggen et al., 2011), supermarket chains (Gaur and Fisher, 2004), vending machine chain (Huang and Lin, 2010), automobile industries (Alegre et al., 2007; Blumenfeld et al., 1987; Ohlmann et al., 2008), meat industry (Oppen et al., 2010), frozen food distribution companies (Custódio and Oliveira, 2006).

Although there are many attributes adapted to the IRP, IRPs are intended to be classified in two schemes. The first one is structural variants, presented by Andersson et al. (2010), in which the number of distributors and customers may vary depending on the nature and essence of the arisen problem. For instance, in maritime logistics, only a central depot does not exist. The ships might load and unload goods in different harbours. The second one is the interested demand characteristics. Due to the nature of the arisen problem, it can be known a priori (deterministic) or uncertain. As for the uncertainty, it can be described random variables, membership functions, or even unknown, if no probability and membership function is used for it.

Topology of the IRP might be one-to-one, one-to-many or many-to-many (Baita et al., 1998). Most previous researches have been on typical versions of IRP with simple topology (one-to-many), single vehicle, single period, single product and operating policy aimed to minimise the combined inventory-routing cost. And also, most researches in the IRP literature have focused on two-echelon distribution systems while distribution centres are tightly connected with the plant. The literature on three-level distribution chains is a little rare. Shen and Qi (2007) conducted a research on a three-level IRP problem with stochastic demands to minimise cost including location cost, inventory cost and distribution cost. Their model determined assigning customers to distribution centres and also the number of location of distribution centres. Zhao et al. (2008) considered a three-level distribution system, in which a single train with a large capacity replenished inventory of the warehouse. Their proposed strategy was the integration of two methods: power of two (POT) for inventory management and fixed partitioning policy (FPP) for transportation. Shen and Honda (2009) focused on a three echelon supply chain including a single plant, multiple distribution centres and multiple retailers and formulated a mixed integer programming (MIP) model to develop an integrated replenishment and routing plan while considering lateral transfers between distribution centres in order to lower transportation cost. Their system was assumed to be operated in JIT replenishment way; therefore, the inventory cost was not incurred to the model. Their heuristic solution approach divided the problem to three sub-problems: in the first one, retailers were assigned to distribution centres and routes were determined for each distribution centre. The remaining two sub-problems corresponded to the transfer between distribution centres and modelling network flow for determining inventory replenishment to minimise costs. Van Anholt et al. (2016) proposed a multi period inventory-routing problem with pickups and deliveries motivated by the replenishment of automated teller machines (ATM) in the Netherlands. Qazvini et al. (2016) presented a mixed integer linear model for a green routing problem. They considered environmental issues in a multi depot distribution system.

Many studies have been carried out on uncertainties in IRP literature. Jaillet et al. (2002) investigated IRP with stochastic demands and infinite planning horizon for a one-to-many network. Ramalhinho Dias Lourenço and Ribeiro (2003) considered a combination of stochastic and deterministic demands for customers simultaneously, i.e., there were two groups of customers: one group had deterministic demand and another had uncertain demands. Some other papers in the IRP literature have focused on stochastic parameters (see, e.g., Adelman, 2004; Yu et al., 2012; Kleywegt et al., 2002; Rahimi et al., 2017). In order to apply any stochastic model to real world problems, a preliminary analysis of uncertainty based on four points is required, which has not yet been performed (Baita et al., 1998):

- a suitably large database of related data
- appropriate assumptions about future behaviour of uncertain parameters
- appropriate assumptions about forms of stochastic processes
- estimation using statistical methods of the parameters of those stochastic processes.

From the practical point of view, such analysis cannot be expected to be easily implemented in every case. So, it seems to be more logical and applicable to consider the unknown parameters as bounds, which leads to reach a more sensible and controllable model.

Robust optimisation is a more recent and novel method for confronting uncertainty, in which the model is set based on an extreme case. There is no available information of parameter distribution and membership function; so, uncertain model is neither stochastic nor fuzzy. As mentioned above, in real life situations, there is not much information about uncertain parameters. Robust optimisation provides a feasible solution (called mini-max solution) under all possible realisation uncertain parameters within their bounds. It means that this approach tries to find solutions which are less sensitive to the changing uncertain parameters (Wang and He, 2009; Gholami-Zanjani et al., 2017).

To the best knowledge of the present authors, the researches done by Aghezzaf (2007) and Solyali et al. (2012) are the only papers which have considered robust optimisation in IRPs. Solyali et al. (2012) assumed an ambiguous probability distribution demand while Aghezzaf (2007) considered normal distribution for both demands and travel times with a constant average and bounded standard deviation called stationary. He developed a nonlinear MIP model for a cyclic distribution strategy. Their proposed model aimed to find minimum cost for transportation and replenishment strategies and the introduced solution approach was a combination of robust plans and Mont-Carlo simulation. Solyali et al. (2012) introduced a one-to-many topology IRP facing uncertainty in demands over a finite time horizon. They proposed two robust MIP models for the problem which were solved by a branch and cut algorithm. For the first time, polyhedral robust approach was implemented within their model.

Since IRP is a NP-hard problem, the model can be only used for exact solutions of small and relatively medium sized problems. Consequently, the intention is to solve the problem via a heuristic method. In the literature, many researchers have implemented heuristic methods for IRPs (see, e.g., Federgruen and Zipkin, 1984; Golden et al., 1984; Anily and Federgruen, 1990). Gaur and Fisher (2004) used a randomised sequential matching algorithm (RSMA) to solve IRP and adapted an insertion method for initial solutions, which was improved by a cross-over method. Campbell and Savelsbergh (2004) obtained initial solutions with an integer programming model and improved it by an insertion method. Zhao et al. (2007) used an insertion method, called Geni, in their research for routing problem and also adapted Tabu search for POT to deal with the inventory problem. Archetti et al. (2017) proposed a Tabu search combined with mathematical formulations to deal with an IRP consisting of a supplier and a set of customers over a discrete time horizon.

This paper is organised as follows: Section 2 describes the problem in detail and the proposed mathematical model. Section 3 presents robust optimisation approach applied to the mathematical model. Section 4 is dedicated to numerical experiments resulted from robust models of the problem. In Section 5, the improved imperialist competitive algorithm is implemented and its controllable parameters are tuned via Taguchi method in Section 6. Section 7 presents some concluding remarks of the paper.

In this paper, a three-level distribution network is introduced, in the first level of which, there is a plant with unlimited capacity of production. The second level consists of a set of distribution centres and, at the third level, there is a set of geographically dispersed customers/retailers served by distribution centres using small vehicles. Since the time horizon is defined as finite and periodic, uncertainty in demands and travelling cost are encountered. In this case, no information is available on uncertainties, except bounds. Corresponding to the most practical distribution management problems, using multiple big vehicles for each distribution centre is allowed, which adds more complexity to the problem. The objective of this problem is to minimise the inventory and routing cost. The replenishment and routing plan is subject to the following constraints:

- the vehicles' capacity cannot be exceeded
- the distribution centres' and customers' capacity cannot be exceeded
- inventory level in the distribution centres and customers' site is not allowed to be negative
- each route starts and ends in distribution centres
- each customer is assigned to just one distribution centre in each time period
- each vehicle is to perform one route per time period from the correspondingly assigned distribution centre
- the number of required vehicles in distribution centres is a decision variable.

2 The proposed mathematical model

Schematic representation of the proposed problem is shown in Figure 1. This graph can be applied to so many real world problems such as food retailing systems, replenishment of automatic teller machines, waste collection systems (Nolz et al., 2014). The problem is defined on a graph $G = (V, A)$, where $V = \{0, \ldots, n\}$ is the vertex set and $A = \{(i, j): i, j \in V, i \neq j\}$ is the arc set. Vertex 0 introduces the plant; vertexes 1, …, d are distribution centres and $(d + 1), \ldots, n$ represent customers. The notations are as follows:

Indices

O plant's index

D set of depots $d = 1, \ldots, D$

C set of depots and customers $i = 1, \ldots, D, D + 1, \ldots, C$

K set of vehicles $k = 1, \ldots, K$.

Parameters

Q_o capacity of vehicles belonging to the plant

Q_k capacity of vehicles belonging to distribution centres

Q_d capacity of distribution centres

Q_c capacity of customers

F_o fixed cost of plant's vehicles

F_k fixed cost of distribution centres' vehicles

d_{it} demand of customer i in period t

C_i capacity of customer i

c_{ij} routing cost associated with going from customer i to customer j

o_d routing cost associated with going from plant to distribution centre d

h_d inventory holding cost rate in each distribution centre

h_i inventory holding cost rate for each customer.

Variables

z_{idt} is equal to 1 if and only if customer i is served by distribution centre d in period t

x_{ijkt} is equal to 1 if and only if vertex j immediately follows vertex i on the route of vehicle k in period t

y_{ikt} is equal to 1 if and only if vertex i is visited by vehicle k in period t

u_{dt} is equal to 1 if distribution centre d receives products from plant in period t

b_{kd} is equal to 1 if vehicle k is allocated to distribution centre d

w_{it} sum of the deliveries made by vehicles, in period t after visiting customer i

q_{ikt} quantity of product delivered to customer i using vehicle k in time period t

g_{dt} quantity of product delivered from plant to distribution centre d in time period t

I_{dt} inventory level of distribution centre d at the beginning of period t

I_{it} inventory level of customer i at the beginning of period t

$$Min\ Z = \sum_t \left(\sum_d h_d I_{dt} + \sum_i h_i I_{it} \right) + \sum_t \sum_k \left(F_o \sum_d u_{dt} + F_k \sum_{i \le |d|} \sum_j x_{ijkt} \right) + \sum_i \sum_j \sum_k \sum_t c_{ij} x_{ijkt} + \sum_t \sum_d u_{dt} o_d$$

$$\sum_d z_{idt} = 1 \quad \forall t, i \tag{1}$$

$$x_{ijkt} \le \frac{1}{3}\left(z_{idt} + z_{jdt} + b_{kd} \right) \quad \forall i \ne j\ \forall t, d \tag{2}$$

$$\sum_{j \ne i} x_{ijkt} = \sum_{j \ne i} x_{jikt} = y_{ikt} \quad \forall i, k, t \tag{3}$$

$$w_{jt} + Q_k \ge w_{it} + Q_k \sum_k x_{ijkt} \quad \forall i, t\ \&\ \forall j > |d| \tag{4}$$

$$q_{ikt} \leq w_{it} \leq Q_k \quad \forall i, k, t \tag{5}$$

$$g_{dt} \leq u_{dt} Q_o \quad \forall d, t \tag{6}$$

$$g_{dt} + I_{dt} = \sum_i \sum_k q_{ikt} b_{kd} + I_{d,t+1} \quad \forall d, t \tag{7}$$

$$\sum_k q_{ikt} + I_{it} = d_{it} + I_{i,t+1} \quad \forall t \,\&\, \forall i > |d| \tag{8}$$

$$I_{dt} \geq 0 \quad \forall d, t \tag{9}$$

$$I_{it} \geq 0 \quad \forall i, t \tag{10}$$

$$g_{dt} + I_{dt} \leq Q_d \quad \forall d, t \tag{11}$$

$$\sum_k q_{ikt} + I_{it} \leq C_i \quad \forall i, t \tag{12}$$

$$\sum_i q_{ikt} \leq Q_k \quad \forall k, t \tag{13}$$

$$q_{ikt} \leq y_{ikt} C_i \quad \forall i, k, t \tag{14}$$

$$\sum_k y_{ikt} \leq 1 \quad \forall i, t \tag{15}$$

$$\sum_i \sum_j x_{ijkt} \leq 1 \quad \forall k, t \,\&\, \forall i \leq |d| \tag{16}$$

$$w_{it}, q_{ikt}, g_{dt} \geq 0 \quad \forall i, k, d, t \tag{17}$$

$$z_{idt}, x_{ijkt}, y_{ikt}, b_{kd} \in \{0, 1\} \quad \forall i, k, d, t \tag{18}$$

In this model, the objective function is to minimise total cost of the system, mainly including two parts of transportation and inventory holding costs. Transportation cost is sum of routing costs from plant to distribution centres and from distribution centres to customers' location along with their fixed costs. Inventory costs are caused by holding inventories in distribution centres and customers' location. These terms are calculated throughout the planning horizon.

Constraint 1 ensures that each customer is allocated to only one distribution centre. Constraint 2 indicates that each vehicle is permitted to visit customers which are allocated to the same distribution centre as vehicles. This constraint avoids the contact in different vehicles' routes belonging to different depots. Flow conservation is guaranteed by constraint 3. Constraints 3, 4 and 5 impose linking conditions and also sub-tour elimination. Constraint 6 means that vehicles' capacities are never exceeded. Constraint 7 balances flow of input and output products in the distribution centres and also define the inventory at the supplier carried at the beginning of the following period. Constraint 8 is similar to 7 but is applied to the customers. Constraints 9 and 10 forbid stock outs in distribution centres and customers, respectively. Constraints 11 and 12 ensure that the amount of product in distribution centres and customers' site should be less than the defined capacity limitation. Moreover, each vehicle is allowed to deliver at most the amount of products equal to its capacity, which is considered in constraint 13. Constraint 14 ensures that, if any product is delivered to a distributor, then there must be at least one vehicle entering that distributor with the vehicle capacity restriction preserved. Constraint 15 states that each customer was allowed to be visited at most once in a period and only by just one vehicle. Constraint 16 accounts for the fact that, in each period, at most one trip can be performed by each vehicle.

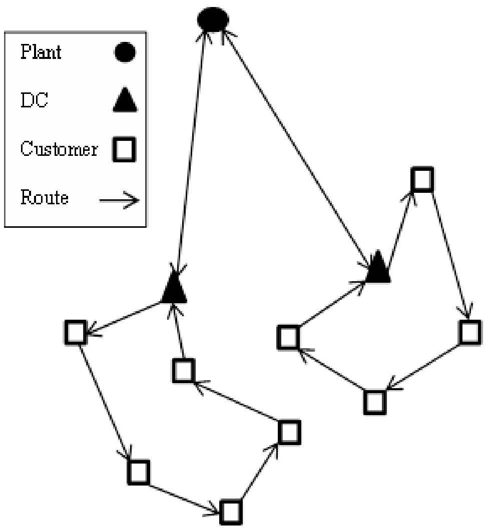
As it is proved, linearisarion immensely reduces computational times. So, an equivalent linear formulation for constraint 7 can be obtained by replacing the term $q_{ikt}b_{kd}$ with auxiliary variable v_{ikdt} and adding the following constraints:

$$v_{ikdt} \leq Mb_{kd}$$

$$v_{ikdt} \geq M\left(1-b_{kd}\right)+q_{ikt}$$

$$v_{ikdt} \leq q_{ikt}$$

Figure 1 Schematic representation of the proposed problem



3 Robust optimisation

Robust optimisation is an important methodology for dealing with data uncertainty in optimisation problems. First, in this method, a deterministic dataset is defined within the uncertain space and then the best solution which is feasible for any realisation of the data uncertainty in the given set is obtained, which is called the corresponding robust counterpart optimisation. In set-induced robust optimisation, the uncertain data are assumed to vary in a given uncertainty set and the aim is to choose the best solution among the ones 'immunised' against data uncertainty, i.e., candidate solutions that remain feasible for all realisations of the data from the uncertainty set.

To briefly illustrate this method in general, consider the following simple linear mathematical model, in which parameters $\tilde{a}_{ij}$ and $\tilde{b}_i$ belong to uncertain interval.

$$\min cx$$
$$s.t. \sum_j \tilde{a}_{ij} x_j \leq \tilde{b}_i \ \forall i$$

The corresponding robust counterpart model is as follows:

$$\min cx$$
$$s.t. \sum_j a_{ij} x_j + \left[\max_{\xi \in U} \left\{ -\xi_{i0} \hat{b}_i + \sum_{j \in J_i} \xi_{ij} \hat{a}_{ij} x_j \right\} \right] \leq b_i$$

where $\tilde{a}_{ij}$ and $\tilde{b}_i$ are replaced with $\tilde{b}_i$ and $b_i + \xi_{i0}\hat{b}_i$, respectively, in which $\hat{a}_{ij}$ and $\hat{b}_i$ are constant perturbation. The related newly added expressions are maximised to deal with the worst case. J_i represents the index subset that contains the variables, the corresponding coefficients of which are subject to uncertainty. ξ_{i0} and ξ_{ij} are random variables subject to uncertainty as well. For the uncertainty set U and for any ξ in the given set, the model has to be immunised against infeasibility.

The box, polyhedral and box-polyhedral uncertainty sets are described by ∞-norm, 1-norm and $\infty \cap 1$ norm of uncertain data, respectively; the data vectors are as follows:

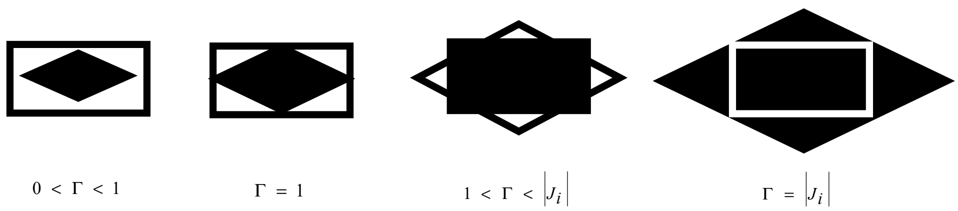
$$U_\infty = \{\xi \,||\, \|\xi\|_\infty \leq \Psi\} = \{\xi \,||\, \xi_j | \leq \Psi, \forall j \in J_i\}$$

$$U_1 = \{\xi \,||\, \|\xi\|_1 \leq \Gamma\} = \left\{\xi \,\middle|\, \sum_{j \in J_i} |\xi_j| \leq \Gamma\right\}$$

$$U_{1 \cap \infty} = \left\{\xi \,\middle|\, \sum_{j \in J_i} |\xi_j| \leq \Gamma, |\xi_j| \leq \Psi, \forall j \in J_i\right\}$$

In which Ψ and Γ are the adjustable parameters to control uncertainty bounds. The illustration of box-polyhedral uncertainty is shown in Figure 2. In fact, combined box-polyhedral uncertainty set is intersection of box and polyhedral sets and does not reduce to neither of them if the adjustable parameters satisfy the following term: $\Psi \leq \Gamma \leq \Psi \, |J_i|$

Figure 2 Illustration of the combined box-polyhedral set



In order to transform the deterministic model to the corresponding counterpart, it should satisfy some formulations. If the uncertainty set is box set, the corresponding robust counterpart constraint is equivalent to:

$$\begin{Bmatrix} \sum_j a_{ij} x_j + \Psi \sum_{j \in J_i} \hat{a}_{ij} x_j \le b_i \\ \left| x_j \right| \le u_j ,\ j \in J_i \end{Bmatrix}$$

Similarly, for polyhedral and box-polyhedral uncertainty sets, the following expressions *a* and *b* should be respectively applied:

$$a) \begin{Bmatrix} \sum_j a_{ij} x_j + \Gamma p_i \le b_i \\ p_i \ge \hat{a}_{ij} \left| x_j \right|,\ j \in J_i \end{Bmatrix} \quad b) \begin{Bmatrix} \sum_j a_{ij} x_j + \Psi \sum_{j \in J_i} w_{ij} + \Gamma z_i \le b_i \\ z_i + w_{ij} \ge \hat{a}_{ij} \left| x_j \right|,\ j \in J_i \\ z_i \ge 0,\ w_{ij} \ge 0 \end{Bmatrix}$$

To represent tractable form of the robust model, objective function and constraint 8 should be converted into their equivalent tractable forms. First of all, the objective function should be transformed as a constraint by the following expressions; Li et al. (2011b) used this transformation in their proposed model.

Considering that constraint 8 is an equation, extreme points of uncertainty set can be extended to convert the equity constraint into its tractable robust counterpart.

Min Z

$$Z \ge \sum_t \left(\sum_d h_d I_{dt} + \sum_i h_i I_{it} \right) + \sum_t \sum_k \left(F_o \sum_d u_{dt} + F_k \sum_{i \le |d|} \sum_j x_{ijkt} \right) + \sum_i \sum_j \sum_k \sum_t \tilde{c}_{ij} x_{ijkt} + \sum_t \sum_d \tilde{u}_{dt} o_d$$

$$\sum_k q_{ikt} + I_{it} = \tilde{d}_{it} + I_{i,t+1} \quad \forall i, t$$

Finally, according to the above descriptions, robust counterparts of the proposed IRP in three-echelon distribution network with regard to three different uncertainty sets (box, polyhedral and combined box-polyhedral) are equivalent to the following MILP problems:

- Box uncertainty set:

  Min Z

  s.t. constraints $1-7$ and $9-18$

$$Z \ge \sum_t \left(\sum_d h_d I_{dt} + \sum_i h_i I_{it} \right) + \sum_t \sum_k \left(F_o \sum_d u_{dt} + F_k \sum_{i \le |d|} \sum_j x_{ijkt} \right) + \sum_i \sum_j \sum_k \sum_t c_{ij} x_{ijkt} + \sum_t \sum_d u_{dt} o_d + \Psi_1 \left(\sum_i \sum_j \sum_k \sum_t G_{c_{ij}} x_{ijkt} + \sum_t \sum_d G_{o_d} u_{dt} \right)$$

$$\sum_k q_{ikt} + I_{it} = d_{it} + \Psi_2 G_{d_{it}} + I_{i,t+1} \quad \forall i, t$$

- Polyhedral uncertainty set:

  Min Z

  s.t. constraints $1-7$ and $9-18$

  $$Z \ge \sum_t \left(\sum_d h_d I_{dt} + \sum_i h_i I_{it} \right) + \sum_t \sum_k \left(F_o \sum_d u_{dt} + F_k \sum_{i \le |d|} \sum_j x_{ijkt} \right)$$
  $$+ \sum_i \sum_j \sum_k \sum_t c_{ij} x_{ijkt} + \sum_t \sum_d u_{dt} o_d + (V_0 \Gamma_1)$$
  $$V_0 \ge \hat{c}_{ij} x_{ijkt} \quad \forall i, j, k, t$$
  $$V_0 \ge \hat{o}_d u_{dt} \quad \forall d, t$$
  $$\sum_k q_{ikt} + I_{it} = d_{it} + \Gamma_2 G_{d_{it}} + I_{i,t+1} \quad \forall i, t$$

- Box-polyhedral uncertainty set:

  Min Z

  s.t. constraints $1-7$ and $9-18$

  $$Z \ge \sum_t \left(\sum_d h_d I_{dt} + \sum_i h_i I_{it} \right) + \sum_t \sum_k \left(F_o \sum_d u_{dt} + F_k \sum_{i \le |d|} \sum_j x_{ijkt} \right)$$
  $$+ \sum_i \sum_j \sum_k \sum_t c_{ij} x_{ijkt} + \sum_t \sum_d u_{dt} o_d + \Psi \left(\sum_{ij} w_{ij} + \sum_d w_d \right) + V_0 \Gamma_1$$
  $$V_0 + w_{ij} \ge G_{c_{ij}} x_{ijkt} \quad \forall i, j, k, t$$
  $$V_0 + w_d \ge G_{o_d} u_{dt} \quad \forall d, t$$
  $$w_{ij} \ge 0 \quad \forall i, j$$
  $$w_d \ge 0 \quad \forall d$$
  $$V_0 \ge 0$$
  $$\sum_k q_{ikt} + I_{it} = d_{it} + G_{d_{it}} \Gamma_2 + I_{i,t+1} \quad \forall i, t$$

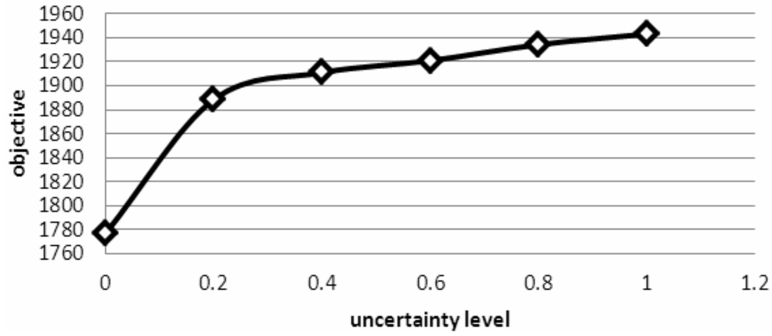
4 Computational results

In this section, some numerical experiments are conducted to assess different viewpoints of robust optimisation performance versus the deterministic version. The input data were randomly generated based on those generated in the research by Li et al. (2011b); locations of distribution centres and customers were randomly generated in a square with size 100 * 100. Coordinates of vendor were [–50, –50]. The customers' demands were uniformly distributed on [0, 20]. The inventory holding cost in the distribution centres and customers was taken as 3 and 6, respectively, and unit transportation costs of the vehicles were taken to be 1. The inventory holding capacities for distribution centres, customers, vehicles belonging to plant and distribution centres were equal to 100, 20, 120 and 54, respectively. Both deterministic and robust models were solved by ILOG CPLEX 10.11 optimisation software and all the tests were carried out on a PC core.i7 1.60 GHz with 4 GB RAM.

To compare performance of the proposed model, the related data were applied to a distribution system with one plant, two distribution centres, ten customers and two time periods. Also, the uncertainty bound covered in this example was 10% of nominal data.

Considering only demand uncertainty, then the only affected constraint was constraint 8, in which the uncertain parameter was RHS of the constraint. Therefore, the number of uncertain parameters was 1 and different uncertainty sets were reduced to 1-dimenstional interval set. Figure 3 shows the results for robust counterparts for different uncertainty levels. As it is shown, the results were identical for the three studied uncertainty sets.

Figure 3 Results for demand uncertainty in different sets and level of uncertainty



Here, only transportation cost uncertainty was considered in objective function, where c_{ij} and o_d are uncertain parameters. Thus, the objective function had 83 uncertain parameters. From the results shown in Figures 4 and 5, it can be seen that, although for $\Gamma \leq 1$, the polyhedral and interval-polyhedral uncertainty sets were equal, for $\Gamma > 1$, the combined uncertainty set resulted in better solutions because the corresponding geometry was less restricted while the polyhedral's solution was sharply deteriorated.

Figure 4 Travelling time uncertainties for polyhedral-interval set

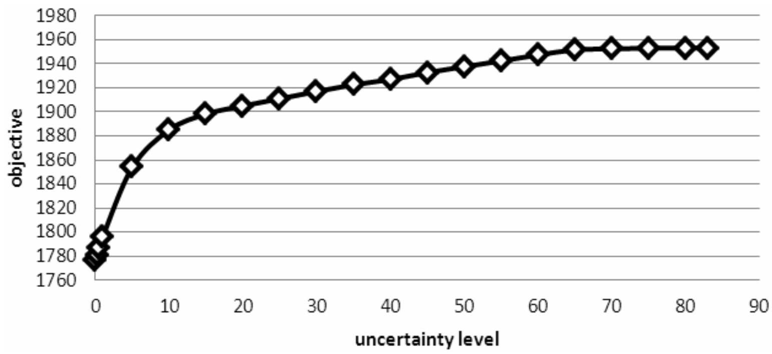
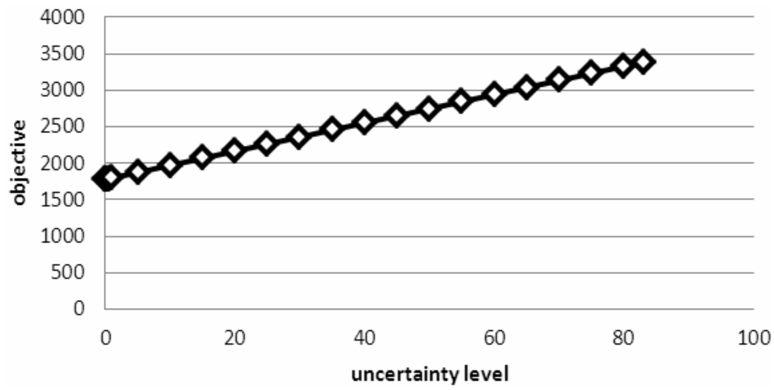
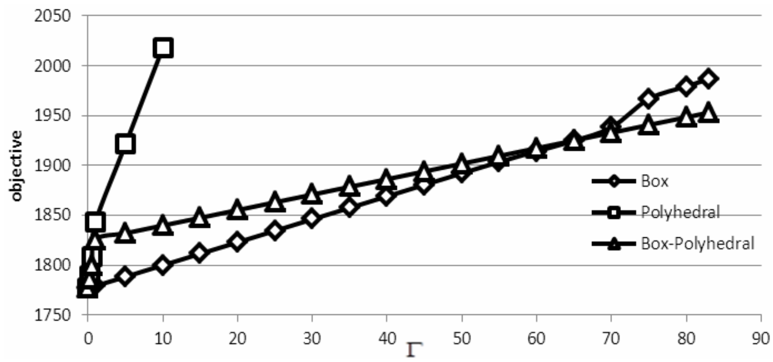


Figure 5 Travelling time uncertainties for polyhedral set



Here, all uncertainties were considered simultaneously: demand and transportation cost. To show results in the same figure using the same axis, $\Gamma_{demand} = \frac{1}{83}\Gamma_{transportation\ cost}$ had to be set. As it can be observed in Figure 6, when Γ, Ψ for demand and transportation cost was zero, the objective functions were equal to those of the deterministic model. According to this analysis, it can be concluded that, to avoid too conservativeness and deteriorated solutions, the interval set should be combined with polyhedral set because the solution for polyhedral set is rapidly deteriorated.

Figure 6 Demand and travelling time uncertainties for three uncertainty sets



5 Improved ICA

Imperialist competitive algorithm (ICA) is an evolutionary algorithm introduced by Atashpaz-Gargari and Lucas (2007). This algorithm uses socio-political evolution of humans as a source of inspiration to develop a strong optimisation method. Since basic ICA is only suitable for problems with continuous variables, the algorithm was improved in order to adapt it to discrete problems. In this case, a crossover and a mutation function of the genetic algorithm were applied to ICA. Below steps of the improved ICA are explained.

5.1 Solution coding (country structure)

A feasible solution proportion to the described model is a three dimensional matrix represented $[M]_{I \times D \times T}$, the dimensions of which are distribution centre (d), time (t) and customer (i). Members of this matrix are the number of products which are delivered from a distribution centre to a customer in each period of time. Term $M_{141} = 15$, for example, means that customer 1 is served by distribution centre 4 in period 1. Members of this matrix should be produced somehow intelligently, not merely randomly in order not to produce infeasible solutions. The pseudo-code for the generation of delivery amount is described in Table 1. Regarding that each customer should be dedicated to just one distribution centre (as shown in Figure 1), just one cell can be a positive number in each row. Moreover, the amount of products transported from each distribution centre should not exceed its capacity. Since lost sales and back orders are not allowed in the proposed problem, in period 1, delivery amounts should be more than demands and less than capacity constraints. For periods 2 to T-1, the generated numbers should be between zero and minimum of constraints. Finally, delivery amounts in the last period are equal to sum of demand in time period minus sum of delivered amounts up to period T-1.

Table 1 Delivery amount pseudo-code

if $t = 1$

$$q_{it} \in \left[d_{it}, \min\left\{ Q_c, Q_k, \sum_{t=1}^{T} d_{it} \right\} \right]$$

for $t = 2: T - 1$

if $I_{it} \geq d_{it}$

$$q_{it} \in \left[0, \min\left\{ Q_c - I_{it}, Q_k, \sum_{t=t}^{T} d_{it} \right\} \right]$$

else

$$q_{it} \in \left[\max\{0, (d_{it} - I_{it})\}, \min\left\{ Q_c - I_{it}, Q_k, \sum_{t=t}^{T} d_{it} \right\} \right]$$

end

if $t = T$

$$q_{it} = \sum_{t=1}^{T} d_{it} - \sum_{t=1}^{T-1} q_{it}$$

After generating each country, sum of the transported products from each distribution centre is known; so, the minimum number of vehicles can be easily calculated. To construct the routes, Clarke and Wright's (1964) saving algorithm was used, which is most widely known in heuristic for the vehicle routing problem and is useful for the problems in which the number of vehicles is a decision variable. This heuristic is based on savings; in the first step, a feasible solution consists of n (number of customers allocated to a DC) direct routes between the depot and customers. Afterwards, two routes are combined into a single route regarding the saving $s_{ij} = c_{0i} + c_{0j} - c_{ij}$. Combinations start from the top of the saving list for every two nodes. This combination is done until there is no further savings.

According to the proposed strategy, initial population with size $N_{country}$ was created. Subsequently, N_{imp} countries were selected from the best members, which meant the ones with the lowest costs of this group were considered the imperialists. Then, there were N_{col} countries left as colonies. Now, the colonies were allocated to imperialists in terms of their power calculated as follows:

$$A = \max\{c_n\} \forall n \in N_{imp}$$

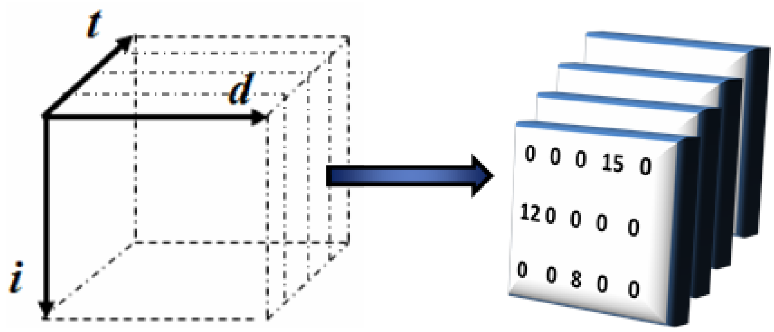
$$C_n = A - c_n$$

c_n is cost of the n^{th} imperialist and C_n is its normalised cost. Cost of each country is determined by fitness function. Also, the imperialists' proportional power is computed as in the following term:

$$p_n = \left| \frac{C_n}{\sum_{n=1}^{N_{imp}} C_n} \right|$$

Consequently, the number of each imperialist's primary colonies is equal to: $NC_n = round\ [p_n \cdot N_{col}]$, in which NC_n represents an imperial's initial number of colonies in the mentioned formula. So, some primary colonies were randomly selected for the n^{th} imperialist and dedicated to it.

Figure 7 Illustration of a country or feasible solution (see online version for colours)



5.2 Solution improvement

5.2.1 Crossover operator

After specifying the imperialists and their colonies, each colony has to move toward its imperialist. To approve it, crossover operation of genetic algorithm was applied to all the colonies by selecting a colony and applying the crossover operation between it and the imperialist. There were two countries as the product of each crossover operation. The lower cost produced country was compared with the colony. If this country had less cost than the colony, it was substituted for the colony; otherwise, both countries were omitted.

To apply the crossover operator, a random number was generated in [0, 1] for each of the customers; if the random number was in [0, 0.5], then, the customer was allocated to the DC corresponding to parent 1; otherwise, it was patterned from parent 2. As mentioned, these numbers should be generated for each customer in each time period. Through this method for crossover, infeasible solutions caused by exceeding the capacity of DCs could be encountered; in this case, a function was used for transforming the infeasible solutions to feasible ones by allocating positive deviations to other DCs which had negative deviation from the capacity.

5.2.2 Mutation operator

To search more space around imperialists, mutation operator, which is another genetic algorithm operator, was used for each colony. To fulfil it, mutation operator was applied to each colony and, in the case of accessing a better result than crossover result, the customer was randomly allocated to another DC which had enough remaining capacity.

5.3 Empire power

According to Atashpaz-Gargari and Lucas (2007), power of an empire is equal to the imperialist's power plus a fraction of its total colonies' power computed as the following term:

$$TC_n = \cos t\left(imperialist_n\right) + \alpha \cdot mean\left(\cos t\left(colonies\ of\ empire_n\right)\right)$$

In which TC_n indicates total cost of an empire and α is a number in interval [0, 1]. In general, $\alpha = 0.2$ is a proper measure and had better results for the proposed problem.

5.4 Imperial competition

In this paper, $\beta = 3$ was considered and the number of the weakest colonies of the weakest empire was possessed by the imperialists' competition. The colonies were proportional to the empires' power, which meant the more the power, the more the possibility. To calculate the probability of each empire possession, considering total cost of an empire was described as follows:

$$B = m\text{ax}\left\{TC_n\right\} \quad \forall n \in \left[1, N_{imp}\right]$$

$$NTC_n = B - TC_n$$

In this formula, TC_n shows the total cost of the n^{th} empire and NTC_n is equal to the normalised cost of that empire, by which possession probability of each empire can be computed using the following term:

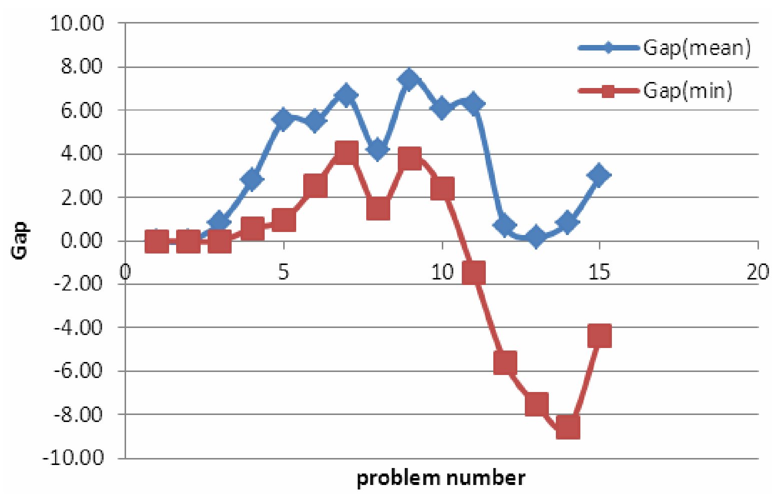
$$p_{p_n} = \frac{NTC_n}{\sum_{n=1}^{N_{imp}} NTC_n}$$

5.5 Eliminating an empire

An empire is eliminated when it has lost all of its colonies. After a while, all the empires, except the most powerful one, will be collapsed. In such a condition, the algorithm is stopped and the only imperialist is displayed as the optimal solution.

Table 2 shows resulted performance of the improved ICA versus the exact method performed by General Algebraic Modeling System Software (GAMS) in order to evaluate effectiveness of the proposed algorithm. Fifteen problems with different sizes were experimented; the column problem info indicates characteristics of the problem in terms of number of plant, number of distribution centres, number of customers and number of planning periods. The meta-heuristic algorithm was run five times for each test problem. 'Average' and 'minimum' of the resulted values were presented in columns and also, to assess the performance of the improved algorithm the gap between the minimum and average value resulted from meta-heuristic's solution and GAMS's solution is defined as: $\%Gap = \frac{ICAz - GAMSz}{GAMSz} \times 100$. Figure 8 shows comparison of the resulted gaps for two criteria. As can be observed, for small sized problems, the solutions were almost the same and the gap was zero; however, with the size increase, the gap increased until violating the time limitation and available memory. The gap decreased to minus values because the exact method was not able to find a proper solution with available resources.

Figure 8 Resulted gaps using exact and meta heuristic algorithms for different problem sizes (see online version for colours)

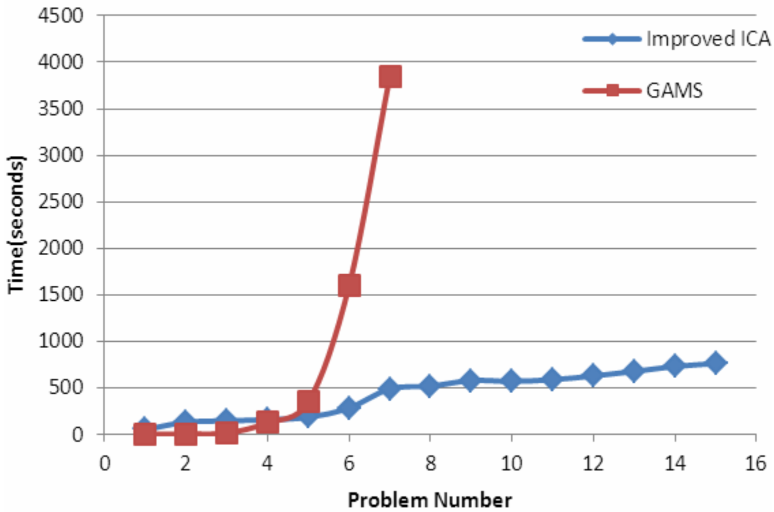


It should be noted that the solutions were computed in two hours because, in more than two hours, the problems caused low memory in the used personal computer. Figure 9 compares solution time of ICA to GAMs, indicating the necessity of hiring and meta heuristic algorithm and showing that average of CPU time is more improved when using the proposed ICA.

Table 2 Comparing the proposed ICA and B&B

No.	Problem info.	Improved ICA objective value		Improved ICA average time	GAMS objective value	GAMS time	Upper bound	%Gap	
	*P*D*C*T*	*Min*	*Average*	*(s)*		*(s)*		Gap^{mean}	Gap^{best}
1	1*1*3*1	447	447	53	447	0.2	529	0	0
2	1*2*5*1	461	461	133	461	6	1,011	0	0
3	1*2*5*2	817	824	146	817	15	2,359	0.86	0
4	1*2*6*2	1,575	1,610	159	1,566	126	2,764	2.81	0.57
5	1*2*6*3	2,210	2,310	185	2,189	350	4,346	5.53	0.96
6	1*1*7*4	2,288	2,355	285	2,232	1,600	4,678	5.51	2.51
7	1*3*12*2	1,385	1,420	490	1,331	3,850	4,942	6.69	4.06
8	1*3*12*3	1,832	1,880	521	1,805	7,200 <	7,326	4.16	1.5
9	1*3*12*4	1,936	2,003	580	1,865	7,200 <	10,325	7.4	3.81
10	1*4*11*3	1,360	1,409	571	1,328	7,200 <	8,052	6.1	2.41
11	1*4*11*4	1,905	2,054	590	1,933	7,200 <	11,736	6.26	–1.45
12	1*4*11*5	2,687	2,699	630	2,846	7,200 <	15,355	–5.17	–5.59
13	1*4*14*4	2,598	2,692	678	2,810	7,200 <	12,399	–4.2	–7.54
14	1*4*16*4	2,716	2,995	732	2,970	7,200 <	12,898	0.84	–8.55
15	1*5*15*4	2,610	2,715	764	2,730	7,200 <	13,948	–0.55	–4.4

Figure 9 Comparing solution time of the proposed algorithm with GAMs (see online version for colours)



6 Parameter designing

The ‘parameter design’, introduced by Taguchi (1987), was used to avoid producing functional variances under external environment influence and to achieve higher robustness. This method puts the controllable factors in inner orthogonal array and noise factors in the outer orthogonal array.

Signal/noise ratio (S/N) is the output obtained through the experiments of measured values of quality characteristics. This method aims to minimise variance of quality characteristics resulted from S/N ratio by tuning the parameters. Additionally, quality characteristics of this research were to minimise total costs; so, the principle “the lower is better” was preferred.

The formula for S/N ratio was derived from the loss function presenting “the lower is better”, as mentioned below:

$$\text{S/N ration: } n_j = 10 \log\left(\frac{\sum_{i=1}^{N} y_i^2}{N}\right)$$

$$\text{Loss function: } L(y) = Ky^2$$

Meta-heuristic parameters considered as controllable factors are mentioned in Table 3, each of the factors might have some levels and each level is for each value. Thus, $4^5 = 1{,}024$ experiments are required for this approach to implement full factorial design. A factorial replicated design is used for considering statistical theories. Since experimenting all combinations of factors was not economical in terms of cost and time, implementing all the experiments was not required.

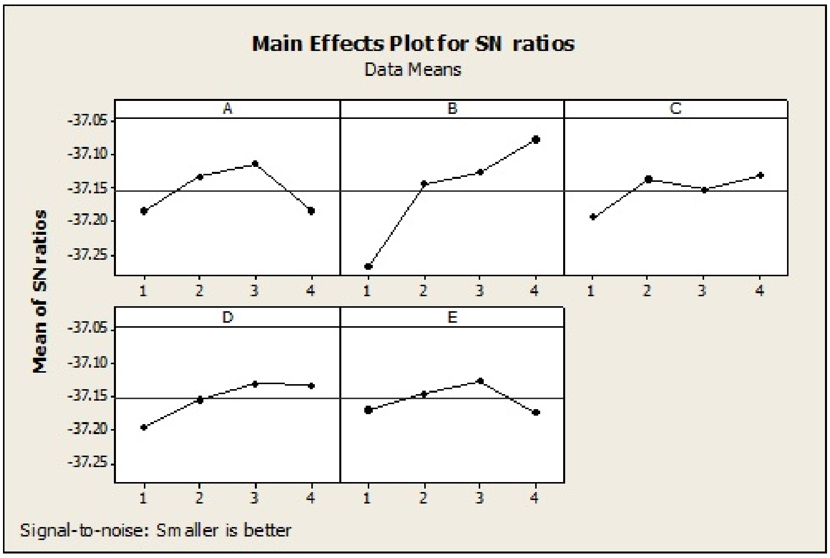
Table 3 Quality characteristics and the considered value for each level

Factors/levels	*1*	*2*	*3*	*4*
Percent of empires	0.1	0.15	0.2	0.25
Number of colonies	80	100	120	140
Crossover percentage	0.7	0.75	0.8	0.85
Mutation percentage	0.1	0.15	0.2	0.25
Mutation rate	0.05	0.1	0.15	0.2

In the next step, control factors were assigned to the columns of the orthogonal array and the corresponding integers in these columns indicated actual levels of these factors. This proper orthogonal array was generated by Minitab software. Note that, in the foregoing scheme, only the main effects were investigated. The results for five replicated trials in each level combination were also solved and, at last, S/N ratios were calculated by the formula mentioned before. The resulted values of S/N ratios are plotted against control factor in Figure 10.

Accordingly, the best combination of control factors in Figure 10 can be deduced; control factors were more effective at their levels 3, 1, 2, 3 and 3 for percent of empires, number of colonies, crossover percentage, mutation percentage and mutation rate, respectively.

Figure 10 Resulted values of S/N ratios are plotted against control factor (see online version for colours)



7 Conclusions

In this paper, robust optimisation models were proposed for three-level inventory routing distribution systems within finite time horizon based on recent extensions in robust optimisation theory. The introduced problem was presented in a MILP model which was different from others in its rich literature. The problem considered a plant, distribution centres and customers at the last level. And depending on periodic demands of customers, they could be assigned to different distribution centres which added more flexibility to the model. The number of vehicles was uncertain and belonged to solutions obtained from the model. Three different techniques of robust optimisation with different uncertainty levels were applied to the mathematical model. The results were compared with each other that helped the decision maker to select the best strategy based on degree of conservatism and vision of the company. Finally, due to discreteness of the proposed model, an ICA was improved using genetic algorithm operators. The numerical results showed effectiveness of the adapted algorithm for different problems, especially for large ones. Also, as it was expected, in three robust techniques used here, the objective function increases as the level of conservatism increases.

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